Spin-charge separation in strongly interacting finite ladder rings

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We study the conductance through Aharonov-Bohm finite ladder rings with strongly interacting electrons, modelled by the prototypical t-J model. For a wide range of parameters we observe characteristic dips in the conductance as a function of magnetic flux, predicted so far only in chains which are a signature of spin and charge separation. These results open the possibility of observing this peculiar many-body phenomenon in anisotropic ladder systems and in real nanoscopic devices.

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I. INTRODUCTION

The phenomenon of the fractionalization of an electron into its spin and charge degrees of freedom was predicted theoretically for one dimensional (1D) strongly interacting systems in the framework of the Luttinger Liquid (LL) theory.[1,2,3] Continuing progress in fabrication techniques, and the discovery of new materials of quasi-1D electronic character, have led in the last decade to a variety of experiments which seek evidence of spin charge separation (SCS) such as the observation of non-universal power-law I-V characteristics,[4] the search for characteristic dispersive features by angle-resolved photoemission spectroscopy (ARPES),[5] the violation of the Wiedemann-Franz law, [6] and the analysis of spin and charge conductivities.[5,7] Among the candidate materials to present SCS[8] we can mention the organic Bechgaard and Fabre salts, molybdenum bronzes and chalcogenides,[4] cuprate chain and ladder compounds[9], laterally confined two-dimensional electron gases, cleaved-edge overgrowth systems[10] and also carbon nanotube systems.[11,12]

From the theoretical point of view, several ways for detecting and visualizing SCS were proposed. Direct calculations of the real-time evolution of electronic wave packets in Hubbard rings revealed that the spin and charge densities dispersed with different velocities as an immediate consequence of SCS.[13,14] The analysis of the electronic transmission through Aharonov-Bohm (AB) rings [15,16,17] described by a LL presented striking features characteristic of SCS, where the flux-dependence of the transmission was found to show new structures appearing at fractional flux values in addition to the non-interacting flux quantum periodicity $\Phi_0 = hc/e$. In[15] these fractions were determined by the ratio between the spin and charge velocities v_s/v_c. In their interpretation the dips arise because transmission requires the separated spin and charge degrees of freedom of an injected electron to recombine at the drain lead after traveling through the ring a different number of turns in the presence of the AB flux. However, recent results which go beyond the single pole approximation used there, suggest that this idea is too simple and claim that the number of dips is not determined approximately by v_c/v_s but by v_J/v_s, where v_J is the current velocity.[16] The results, however, agree for small integer values of p and q, for $v_s/v_c = p/q$. Recent numerical calculations of the transmittance through finite AB rings described by the t-J model show clear dips at the fluxes that correspond to the ratio v_s/v_c.[17] As we explain below, the discrepancy arises due to the finiteness of the system.

In spite of the clear indications of the existence of spin-charge separation in 1D interacting systems and its absence in three dimensions where the Fermi Liquid theory is valid, there is no final word for two dimensions. The non-Fermi-liquid normal state properties of high temperature superconductors have led to attempts to trace their origin in the possible realization of SCS in strongly correlated electron systems in 2D.[18]

In this paper we explore the possibility of the existence of SCS in ladders, as a first step towards two dimensions. We analyze the conductance through rings formed by two-leg ladder systems described by the t-J model as a prototype of interacting systems. For certain parameters we find, indeed, clear dips at fractional values of the magnetic flux which we can interpret as fingerprints of charge and spin separation due to the difference in the charge and spin velocities.

II. THE MODEL

Our model Hamiltonian reads $H = H_{\text{leads}} + H_{\text{link}} + H_{\text{ring}}$, (Fig. 1), where H_{leads} describes free electrons in the left and right leads,

$$H_{\text{link}} = -t' \sum_{\sigma} (a^{\dagger}_{-1,\sigma} c_{0_1,\sigma} + a^{\dagger}_{1,\sigma} c_{L/2_1,\sigma} + \text{H.c.}) \quad (1)$$

describes the exchange of quasiparticles between the leads ($a_{i,\sigma}$) and particular sites of leg 1 ($c_{i_1,\sigma}$), and

$$\begin{aligned} H_{\text{ring}} &= -eV_g \sum_{i,l,\sigma} c^{\dagger}_{i_l,\sigma} c_{i_l,\sigma} - t_{\parallel} (c^{\dagger}_{i_l,\sigma} c_{i_l+1,\sigma} e^{-i\phi/L} + \text{H.c.}) \\ &- t_{\perp} \sum_{i,\sigma} (c^{\dagger}_{i_1,\sigma} c_{i_2,\sigma} + \text{H.c.}) + H_{\text{int}} \end{aligned} \quad (2)$$

describes the interacting electron system. The fermionic operators $c^{\dagger}_{i_l,\sigma}$ create an electron at site $i = 1, L$ of leg

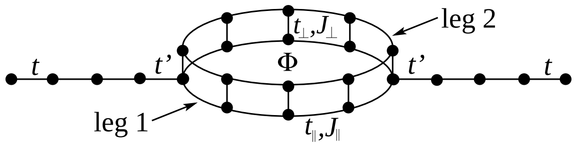

FIG. 1: Schematic representation of the system.

$l = 1, 2$ with spin σ. The AB ring has L rungs, is threaded by a flux ϕ ($\phi = 2\pi\Phi/\Phi_0$), and is subjected to an applied gate voltage V_g.

As we will consider the t-J model for the ladder, the interacting part of the Hamiltonian reads:

$$H_{\mathrm{int}} = J_\parallel \sum_{i,l} \mathbf{S}_{i_l} \cdot \mathbf{S}_{i_l+1} + J_\perp \sum_i \mathbf{S}_{i_1} \cdot \mathbf{S}_{i_2} \tag{3}$$

where $\mathbf{S}_{i_l} = \sum_{\alpha\beta} c^\dagger_{i_l\alpha} \sigma_{\alpha\beta} c_{i_l\beta}$ is the spin at site i and leg l and no double occupancy is allowed.

When the ground state is non-degenerate, the zero temperature transmission from left to right can be calculated to second order in t' by means of the retarded Green function for the isolated ladder ring between sites i and j: $G^{\mathrm{R}}_{i,j}(\omega)$, for an incident particle with energy ω and momentum $\pm k$:[15,19]

$$T(\omega, V_g, \phi) = \frac{4t^2 \sin^2 k |\tilde{t}(\omega)|^2}{|[\omega - \epsilon(\omega) + te^{ik}]^2 - |\tilde{t}^2(\omega)||^2}, \tag{4}$$

where $\epsilon(\omega) = t'^2 G^{\mathrm{R}}_{0_1,0_1}(\omega)$, the effective hopping across the ring is $\tilde{t}(\omega) = t'^2 G^{\mathrm{R}}_{0_1,L/2_1}(\omega)$, and $\omega = -2t\cos k$ is the tight-binding dispersion relation for free electrons in the leads. This equation is in fact exact for a non-interacting system; with interactions on the ring it serves as an approximation in the tunneling limit $t'/t \ll 1$[15,19] for a non-degenerate ground state. For an odd number of electrons, the ground state is Kramers degenerate for a system with time reversal symmetry and the equation ceases to be valid.[19,20] So we assume that the ensuing Kondo effect is destroyed by temperature or magnetic field.[17,19] The conductance is $G = (2e^2/h)T$.

We calculate $T(\omega, V_g, \phi)$ by numerically diagonalizing the isolated interacting ring in the presence of a magnetic field with L rungs and N electrons in the ground state to obtain the Green functions which appear in Eq.(4), fixing the chemical potential of the non-interacting leads to zero ($\omega = 0$). By varying V_g, $T(0, V_g, \phi)$ presents narrow peaks with a width proportional to $(t')^2$ at gate voltages which correspond to the excitation energies of the system with $N-1$ particles.[17,19] The transmittance is obtained by integrating the spectra over a small energy window at the Fermi energy.[15,16,17]

III. RESULTS

In order to study the robustness of the spin-charge separation in the presence of a second chain, we first show results for weakly coupled chains ($t_\perp \ll t_\parallel$) and $J_i = 0$ ($i = \perp, \parallel$), for which we know that in one chain there is complete SCS.[17,21,22] In Fig. 2 we show the results for several small values of $t_\perp$ and in fact, observe clear dips at certain fractional values of the magnetic flux (the abrupt jumps correspond to other level crossings). This is the first evidence, up to our knowledge of charge-spin separation in finite systems with more than one chain.

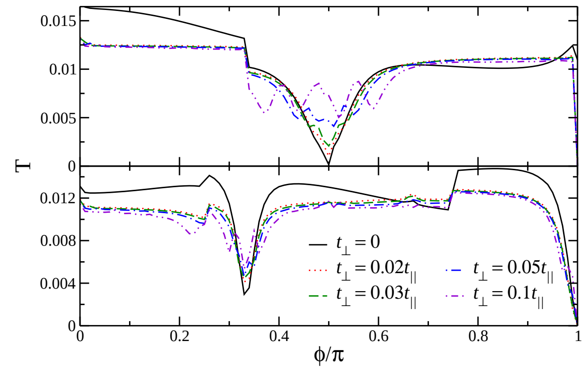

FIG. 2: Weakly coupled chains: Transmittance as a function of flux for the anisotropic t-J model with $J = 0$, $t_\parallel = 0.1$, $L = 6$ rungs, $t' = 0.05t_\parallel$, several values of $t_\perp$ and $N = 6$ (top) and $N = 8$ (bottom) electrons in the ground state.

To understand the position of the dips we resort to the expression obtained in 1D for $J = 0$ ($i = \perp, \parallel$)[17,23]. Considering a non-degenerate ground state containing $N = N_e + 1$ particles and analyzing the part of the Green function that enters the transmittance when a particle is destroyed, it is shown that the dips occur when two intermediate states cross at a given flux and interfere destructively. These particular fluxes depend on the spin quantum numbers, and are located at

$$\phi_d = \pi(2n+1)/N_e, \tag{5}$$

with n integer. If the integration energy window includes these levels, a dip in the conductance arises.

For the ladder with $t_\perp = 0$ and a total even number of electrons N in the ground state, the lowest-lying state has $N/2$ electrons in each leg. As we are calculating the transmittance through one leg only, and the intermediate state has one particle less, from the condition for ϕ_d with $N_e = N/2 - 1$, one expects to see dips at $\phi_d = \pi\frac{2n+1}{N/2-1}$. In Fig. 2 we see that this is the case since for the top figure there will be $N_e + 1 = 3$ electrons in each leg, leading to a dip at $\phi = \pi/2$ and for the bottom figure there will be 4 electrons in each leg leading to a dip at $\phi = \pi/3$. When the "second" dimension is turned on and $t_\perp \neq 0$, we find that the dips remain and are quite robust, even for values of $t_\perp/t_\parallel$ as high as 0.1.

These are so far the results for a weak interchain coupling. Another atracting case is the one with a large coupling between the legs, $t_\perp \gg t_\parallel$. In this limit and for the noninteracting case, the bands corresponding to the bonding and antibonding states of each rung are very far

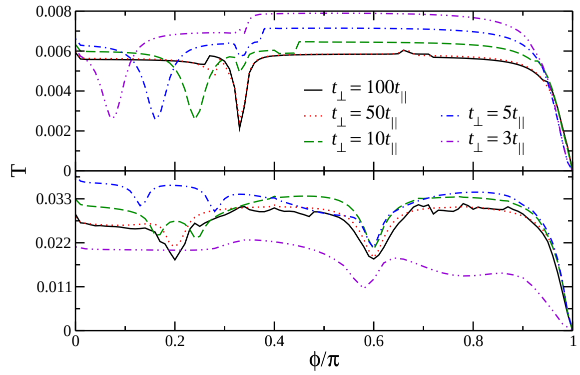

FIG. 3: Strongly coupled chains: Flux-dependent transmittance for $L = 6$ rungs, $J = 0$, $t_\parallel = 0.1$. Top: $t' = 0.05t_\parallel$ and $N = 4$ electrons. Bottom: $t' = 0.09t_\parallel$ and $N = 6$ electrons.

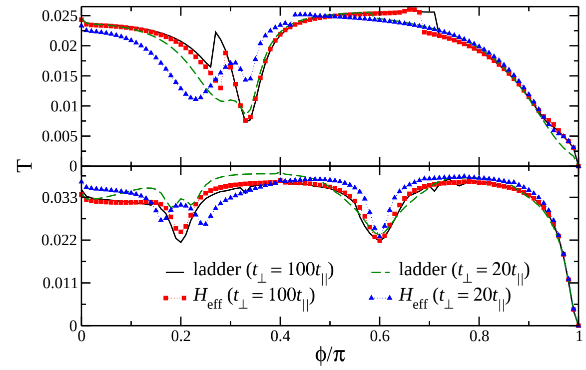

FIG. 5: Transmittance through a bonding channel and comparison with $H_{\rm eff}$ for the same system as in Fig. 3.

apart and one might expect the reappearance of SCS. In fact, this is the case, as can be seen in Fig. 3, where we plot the transmittance for a ladder with several values of $t_\perp$ and fillings. Now the total number of electrons in the lower band corresponds to the total filling N (for a less than half filled band) and the transmittance will involve $N_e = N - 1$ electrons. Hence, if SCS exists, the dips will be found at fluxes $\phi_d = \pi\frac{2n+1}{N-1}$. In this figure we find that for large values of $t_\perp/t_\parallel$, the dips correspond indeed to these fluxes. For smaller values of $t_\perp$, we find a shift in the location of the minima and sometimes a splitting of the dips.

It is interesting to visualize the behaviour of the dips when changing the parameters from weakly interacting chains to the strong coupling case (small to big $t_\perp/t_\parallel$). In Fig.4 we collect the data for $N = 6$ particles, where we see that for $t_\perp/t_\parallel \ll 1$ only one dip shows up (reflecting the bahaviour of half the number of particles in each band corresponding mainly to each leg of the ladder as in Fig. 2). On the contrary, for $t_\perp/t_\parallel \gg 1$ all N particles belong to the relevant lower (bonding) band and two main dips arise at the positions corresponding to this strongly coupled case as in Fig. 3.

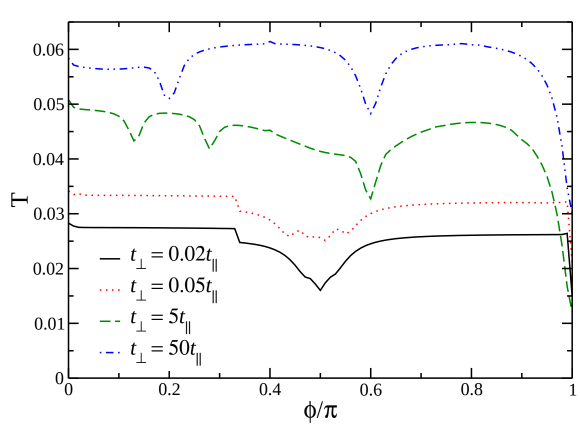

FIG. 4: Transition from weakly-coupled to strongly-coupled chains: Flux-dependent transmittance for several relations of inter to intrachain hoppings $t_\perp/t_\parallel$ for $L = 6$ rungs and $N = 6$ particles.

IV. EFFECTIVE MODEL FOR STRONGLY COUPLED CHAINS

In order to study the strongly coupled case and understand the transition towards the limit of isotropic exchange, we have mapped our ladder Hamiltonian $H_{\rm ring}$ to an effective model in the subspace of the bonding states of each rung, using degenerate perturbation theory up to second order in $t_\parallel$.[24] The model is valid for energies lower than $t_\perp$ and a less than half-filled system:

$$\begin{aligned} H_{\rm eff} &= -t_\parallel \sum_{i\sigma}(\hat{c}^\dagger_{i\sigma}\hat{c}_{i+1,\sigma} + {\rm H.c.}) + J\sum_i(\mathbf{S}_i\cdot\mathbf{S}_{i+1} - 1/4) \\ &\quad + t'' \sum_{i\sigma}(\hat{c}^\dagger_{i+2,\sigma}\hat{c}_{i\sigma}(\mathbf{S}_i\cdot\mathbf{S}_{i+1} - 1/4) + {\rm H.c.}), \qquad (6) \end{aligned}$$

where $J = J_\parallel/2 + 2t_\parallel^2/(t_\perp - 3J_\perp/4)$, $t'' = t_\parallel^2/(t_\perp - 3J_\perp/4)$ and $\hat{c}_{i\sigma} = \frac{1}{\sqrt{2}}(c_{i_1,\sigma} + c_{i_2,\sigma})$, the bonding operator.

The second dimension of the original ladder is reflected by the second nearest-neighbour term which tends to destroy the dips. In Fig. 5 we show the transmittance of a particle through a bonding channel and compare it with that of $H_{\rm eff}$, finding excellent agreement for the strongly coupled case. As $t_\perp/t_\parallel$ diminishes, the curves start to differ as $H_{\rm eff}$ loses its validity. Comparing with Fig. 3, where a particle is injected to one site only instead of into a bonding state, we find that the effective model is also valid for this case, as long as $t_\perp \gg t_\parallel$, since the antibonding band is shifted to upper energies.

So far we have presented results for $J_\parallel = J_\perp = 0$. Finite interactions introduce an extra spin shuffling in the system, mixing the spin wave quantum numbers. In spite of the fact that the conditions that lead to Eq.(5) (based on the $J = 0$ limit) are no longer valid, dips are still observed for small values of the interactions in the strongly coupled case. However, the effect of the interaction is to

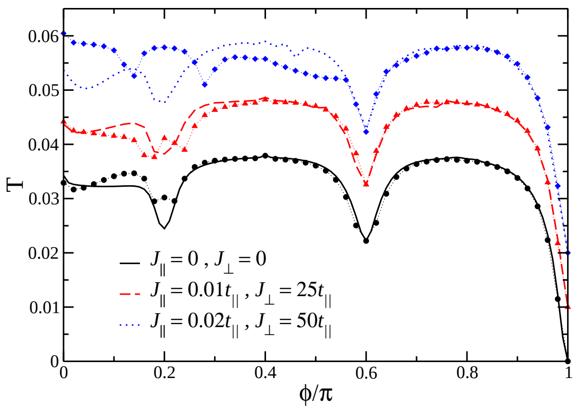

FIG. 6: Transmittance through a bonding channel (lines) and comparison with $H_{\rm eff}$ (symbols) for finite interactions $J_{||}$ and $J_\perp$ for $t_\perp = 50t_{||}$, $L = 6$ rungs and $N = 6$ electrons.

reduce the depth of the dips and shift their position. In Fig.6 we show results for the 6-rung ladder with $N = 6$ particles in the ground state and several values of the interactions. Taking into account the fact that the J's are obtained perturbatively from the large-U Hubbard model, we keep their relation as $J_\perp/J_{||} = t_\perp^2/t_{||}^2$. Two observations can be made: i) The dips are still present for finite J's. ii) However, as for the $J = 0$ case where the dips were affected by the interchain hopping parameter, in this case we also find shifts and reductions in the their depth caused by the interactions. A similar behaviour occurs for weakly interacting chains. We also find that the effective model fits quite well the results in the ladder for finite $J_\perp$ and $J_{||}$, and its range of validity extends to appreciable values of the interaction parameters.

V. CONCLUSIONS

In summary, we have found that the dips in the conductance, predicted to appear in strongly correlated chains as a consequence of spin-charge separation, are robust in the presence of a second transmission channel modelled by a ladder system in the anisotropic limit. For a wide range of parameters, in particular for weak and strong hopping couplings across the rungs $t_\perp$, the dips remain. However, their position differ from the predictions stemming from the exactly solvable case of the Hubbard chain with infinite U (or t-J model with $J = 0$).[17] For intermediate values of $t_\perp$ the dips disappear. The signatures of spin-charge separation are also robust for finite, albeit small, values of spin-spin interactions. This is the first time in which signatures of spin-charge separation are observed in interacting finite systems with more than one chain, thus opening the possibility of measuring this peculiar phenomenon in real nanoscopic systems.

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